LABORATORY FOR PRODUCTION MEASUREMENT

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SOP 23

CALIBRATION OF TAPE MEASURES

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Evidence of changes (regarding the previous issue)

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	Chapter	Chapter	Chapter
1	10.2		
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CONTENTS

1 INTRODUCTION

This procedure describes steps taken when calibrating different kinds of tape measures up to 200 m. Measures are of different design, what influence the way of fixing during the calibration. Some tape measures have prescribed tracting force by which they shall be loaded during calibration.

1.1 General

Tape measures (in the following text "measures ") are simple instruments comprising scale-marks whose distances are given in legal units of length. They can be used in legal and non-legal metrology. The procedure concerns all kinds of measures. Different kinds of measures of length are defined by the Directive 73/362/EEC [2] and are described in the procedure. Regarding the accuracy, these measures are divided into classes I, II in III.

2 MEASURING EQUIPMENT

- Universal length measuring device — Merilnik dolžin 13M
- Video probing system (VPS) — DM
- Laser interferometer (*LI*) — HP 5528 A
- weights 0,5 kg, 1 kg, 2 kg in 5 kg — Utež Celje

3 RECEIPT

The tape measure is received from the customer and is visually checked for any obvious defects like scratches or other defects (e.g. corrosion), which would impede the calibration. The customer’s name, the type and the identification number of the scale are also noted. It is checked, whether the number of received measures in in conformance with the relevant documentation.

4 CLEANING

The line scale is cleaned with a dry cloth. Tougher stains are cleaned alcohol. At this occasion it is checked if graduation and numbering is complete, readable and undeletable.

5 TEMPERATURE STABILISATION

The line scale is stabilised at 20 ^{o}C ± 1 ^{o}C for 24 hours.

6 CALIBRATION

6.1 Visual check

The line scale is visually checked for any obvious defects like scratches or other defects (e.g. corrosion), which would impede the calibration.
The customer should be informed if the calibration due some damage cannot be carried out. The consent should be obtained from the customer if some standard or calibration procedure (eg. change of measuring points, increased uncertainty) should be changed due to some anomalies.
Any anomalies (damage, scratches, damage record,...) should be recorded in the calibration record (Chapter 8).

6.2 Measurement procedure

6.2.1 Terms and explanations

Video screen: Screen, where the picture from the video camera is shown.
Measurement window: Rectangular sector within the video screen, defined using a mouse. Only one scale mark is allowed in the measurement window. Measurement window shall be as wide as possible, but not exceeding the length of the scale interval. Both horizontal edges of the window must be as far apart as possible, but shall not include mark's ends and parts of marks, which are of poor quality.
Reference line: Vertical blue line, that designates the annulled position of the video system.

6.2.2 Fixing the tape measure

A line scale is put on the measurement base like shown in Figure 1.

Figure 1: Fixture of line scales

The measure of length is fixed in accordance with the kind of the measure (point 6.1) in a special fixture. On the other end it is loaded (when necessary) with a proper weight in order to achieve appropriate tension of the material measure.

a) Tape measures with fixed tongue or with sliding tongue for measuring inside and outside dimensions (paragraphs 9.1 and 9.4.1 EEC) are fixed in special fixture as shown in Figure 1. Fixture for checking deviations in the position for measuring inside dimensions is shown in Figure 1 a, and in the position for outside dimensions in Figure

1 b. If the measure of length is longer than 10 m, it is measured in more steps. In the second and in the following steps the measure is fixed with clamps. On the free hanging end the measure is loaded with a weight that corresponds to the defined tractive force (2 kg for measures in paragraph 9.1 EEC). If tractive force is not prescribed, the measure is loaded with the weight 0,5 kg. Figure 2 shows the way of fixing the weight.

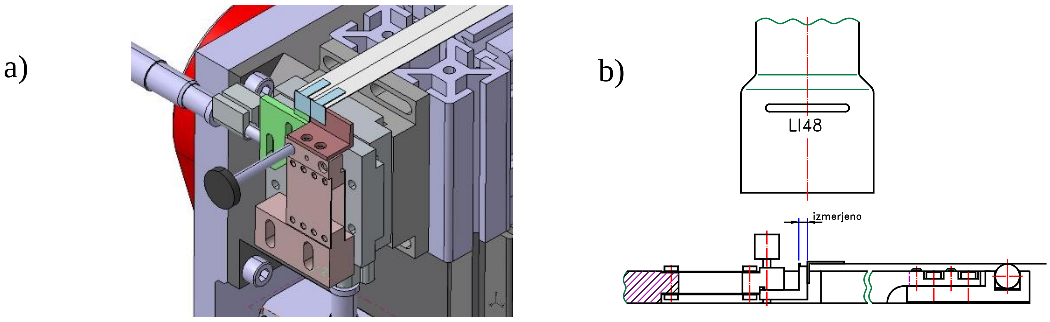

Figure 1: Fixture of measures with fixed or sliding tongue

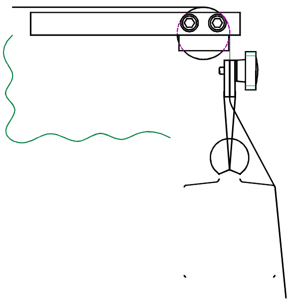

Figure 2: Fixture of the weight

b) Tape measures with hooks, rings or handles (paragraph 9.4.2 in 9.4.3 of EEC) are fixed as shown in Figure 3. End surface of end or composite measures is set into zero position using special part (Figure 3). Since these measures are always longer than 10 m, they are measured in steps as described in a). On the free hanging end the measure is always loaded with a weight specified on the measure tape. (Figure 2).

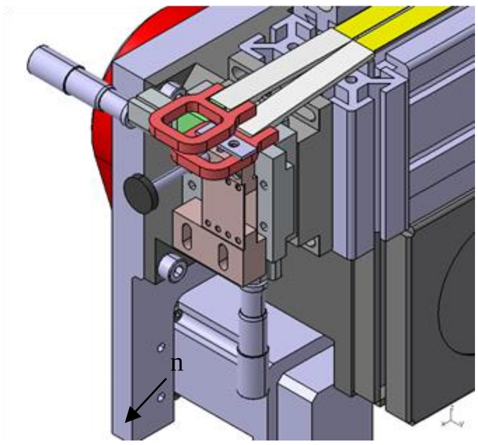

Figure 3: Fixture of measures with hooks, rings or handles

d) Composite metal dip-tapes with sinkers are fixed as shown in Figure 4. Since these measures are always longer than 10 m, they are measured in steps as described in a). On the free hanging end the measure is loaded with a weight of 0,5 kg or 1 kg (depending on the marked mass on the sinker) as shown in Figure 2.

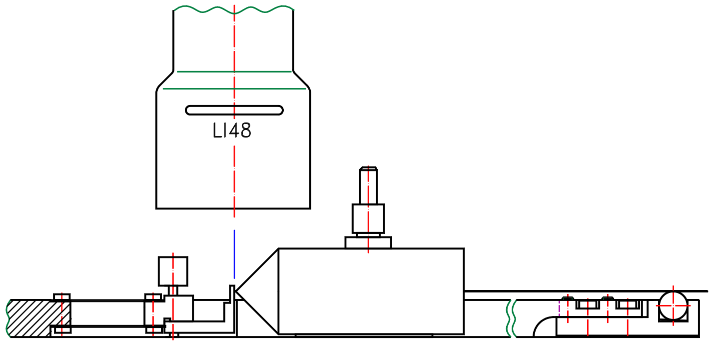

Figure 4: Fixture of the dip-tapes with sinker

6.2.3 Adjustment of the measure of length

Before measurement the measure of length shall be adjusted parallel to the axis of measurement. Measure of length shall be leaned on the back of the profile. The beginning of the measure of length is positioned by the linear micrometre stage in the vertical and horizontal direction.

6.2.4 Setting the signal level

The level of the signal defines the difference in the brightness between the background and the scale mark. To set the level of the signal, the measurement window has to be drawn (Figure 5). The level setting can be automatic (AUTO ON) or manual (AUTO OFF). The scale mark's color must be set by dark/bright switch (TEMNA / SVETLA) (Figure 6).

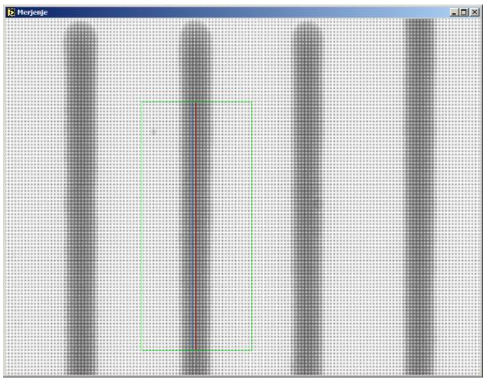

Figure 5: Video screen with measurement window

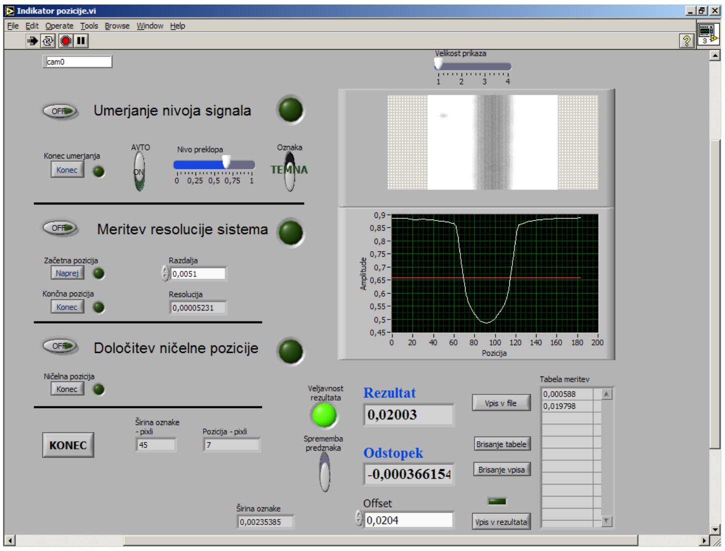

Figure 6: User interface of the video probing system

6.2.5 Setting the origin point

Draw the measurement window approx. in the middle (horizontally) of the video display and position the origin mark within (approx. in the middle). Complete the signal level setting by clicking the button “KONEC”. Complete the determining the resolution with button “OFF”. With button “ON” the setting of the origin point is enabled. Complete the origin point by clicking the button “KONEC”. Blue reference line in video screen appears.

6.2.6 Measurement

Distances between the zero- mark and 10 to 30 random scale-marks including the end-mark are measured. The number of measuring points depends on the length of the scale. The measurement is repeated 3 times in each measurement point.
If the zero point is defined by an end surface, the reference scale-mark for the measurement (scale-mark from which distances are measured) is the closest mark indicating a rounded value (e.g. 10 mm).

Every value is measured five times. Measured values are recorded into the measurement record (see Chapter 8).

6.2.6.1 Measurement with incremental measuring system

Read the position value from the incremental encoder (Figure 7), when the red line covers the blue line in the video screen (Figure 8).

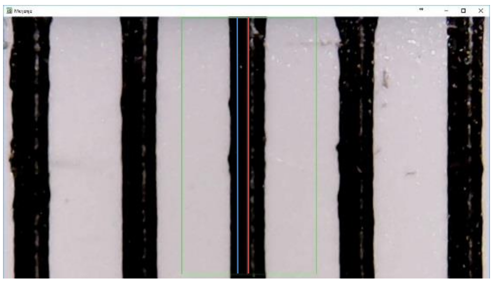

Figure 7: Adjusting the position when measuring the distances between markings

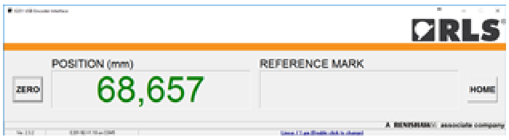

Figure 8: The incremental encoder value

6.2.6.2 Measurement on measurement system with laser interferometer

Read the position value from the laser interferometer (Figure 9), when the red line covers the blue line in the video screen (Figure 7).
In the program the LI the temperature of the material must be set to 20 ° C, whereas the temperature correction is performed subsequently in the measurement record (see Chapter 8).

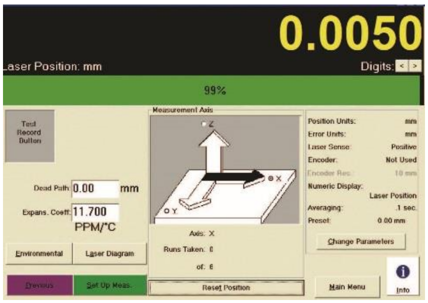

Figure 9: The laser interferometer value

6.2.7 Correction of the measurement result

<u>Correction of thermal expansion</u>
Temperature is measured on the base plate in two points using the contact sensors of HP 5528 A. Measured temperatures are written in the measurement record (see Chapter 8).

<u>Correction of Abbe error</u>
Error caused by the pitch of the moving part of the device carrying VPS and *LI* optics is corrected on the basis of the angle measured by the *LI*. The relative angles were measured in the fix points. The result (relative angle) is written into the EXCEL programme (see Chapter 8), which calculates Abbe error correction using the results of the test in which we measured influence of the pitch angle of the moving part on the change of measurement result.

<u>Correction of LI indication</u>
The results of the specific items must be corrected for the value represented by systematic deviations of parameters (temperature, pressure). This correction is carried out on-line in the program LI.

<u>Correction of incremental system indication</u>
The results of the specific positions must be corrected for the value represented by systematic measurement deviations according to the calibrated value of the incremental position measuring system. The calibration results are written into the measurement record (see Chapter 8).

7 EVALUATION OF CALIBRATION RESULTS

Test calibration results are written into the measurement record (see Chapter 8), which calculates the final calibration results.

8 DOCUMENTATION

The calibration record is produced by using a template »Predloga_zapis-SOP23.xltx«, which is stored in the folder »4-Sistem kakovosti/SOP/SOP23-Tračna merila «. The record is sored into a relevant sub-folder (SOP 23/Zapisi, relevant year) of the folder »3-Zapisi in certifikati«. Instruction for storing records in Quality manual (Ch. 6.2) shall be followed. The second page of the calibration certificate is created by using a template »Predloga_2stran-SOP23.dotx«, which is stored in the folder »4-Sistem kakovosti/SOP/SOP23-Tračna merila «. The calibration certificate is created and archived in accordance with instructions in the Quality Manual (Ch. 6.1).

9 PROTECTION

No special protection is required.

10 MEASUREMENT UNCERTAINTY – LASER INTERFEROMETER

10.1 Mathematical model of measurement

Deviation e (measurement result) is given by the expression:

$$e = L_\mathrm{m} \cdot \left(1 + \alpha_\mathrm{m} \cdot \theta_\mathrm{m}\right) - L_\mathrm{LI} + e_\mathrm{cos} + e_\mathrm{mp} + e_\mathrm{a} + e_F \quad (1)$$

where:

- e - deviation (measurement result) at 20 °C
- L_m - path length between the reference position of the video probing system and the measurement position of the video probing system
- α_m - linear temperature expansion coefficient of material measure of length
- θ_m - temperature deviation of the measure of length from 20 °C
- L_LI - corrected length shown by *LI*
- e_cos - cosine error of measurement (supposed to be 0)
- e_mp - dead path error (supposed to be 0)
- e_a - Abbe error caused by angular deviation of the video probing system (supposed to be 0)
- e_F - error due to the tractive force

10.2 Standard uncertainties of the estimations of the input values and combined standard uncertainty of measurement

Equation for calculated combined standard uncertainty [3,4] has the following form:

$$u_c^2(e) = c_{Lm}^2 u^2(L_m) + c_{\alpha m}^2 u^2(\alpha_m) + c_{\theta m}^2 u^2(\theta_m) + c_{Li}^2 u^2(Li) + c_{ecos}^2 u^2(e_{cos}) + c_{emp}^2 u^2(e_{mp}) + c_{ea}^2 u^2(e_a) + c_{eF}^2 u^2(e_F) \quad (2)$$

where c_i are partial derivatives of the function (1):

$c_{Lm} = \partial f/\partial L_m = 1+\alpha_m \cdot \theta_m \approx 1$; če je $\theta_{mmax}=\pm 1°C$

$c_{\alpha m} = \partial f/\partial \alpha_m = \theta_m \cdot L_m \approx \theta_m \cdot L = 1\ °C \cdot L$ for presumed measurement conditions (CMC)

$c_{\theta m} = \partial f/\partial \theta_e = \alpha_m \cdot L_m \approx \alpha_m \cdot L = 10^{-5}\ °C^{-1} \cdot L$ for steel (in all cases)

$c_{Li} = \partial f/\partial L_i = -1$

$c_{ecos} = \partial f/\partial \delta e_{cos} = 1$

$c_{emp} = \partial f/\partial e_{mp} = 1$

$c_{ea} = \partial f/\partial e_a = 1$

$c_{eF} = \partial f/\partial e_F = 1$ L – nominal length

Standard uncertainties of influence values are evaluated (estimated) for the applied standard, calibration procedure, and presumed measurement conditions. In the CMC evaluation, we presume a tape measure with 4 μm repeatability, and thermal expansion coefficient of $10^{-5}\ °C^{-1}$ (steel). Uncertainty of an actual calibration shall be evaluated by using the Excel file »Negotovost-SOP23.xlsx« (stored in the folder »SOP 23-Tračna merila«). Repeatability, and actual material of the tape measure under calibration are taken into account.

a) ***Uncertainty of the path length between the reference position of the video probing system and the measurement position of the video probing system*** $u^2(L_m)$

The uncertainty is composed of the positioning uncertainty in the reference point $(u(poz_{ref}))$ and of the positioning uncertainty in the measurement point $(u(poz_{mer}))$. It is supposed that both uncertainties are equal $(u(poz_{ref})) = (u(poz_{mer})) = (u(poz))$. Total uncertainty is:

$$u(L_m) = \sqrt{u(poz)^2 + u(poz)^2} = u(poz) \cdot \sqrt{2}$$

Standard positioning uncertainty was determined according to the resolution of the incremental system which is $1\,\mu m$. Standard deviation of these measurements that is accepted as standard uncertainty was:

$$s = (u(poz)) = 0{,}5\ \mu m;$$

Due to the quality of the lines of the tape measures 4 μm was added.
Therefore:

$$u(L_m) = 0{,}71\ \mu m + 4\ \mu m$$

Furthermore, if the measure is longer as 10 m, it should be measured in more steps.

When repositioning the measure, additional positioning uncertainty appear. For 200 mm, this component is:

$$u\left(L_{\mathrm{m}200}\right)=\left(u\left(poz\right)\right)\cdot\sqrt{200/10}=2{,}2\ \mu\mathrm{m}$$

b) *Uncertainty of linear temperature expansion coefficient $u(\alpha_\mathrm{m})$*

Because scales can be made of different materials, a deviation interval of $\pm 2\cdot 10^{-6}\ °\mathrm{C}^{-1}$ is estimated. Standard uncertainty at supposed rectangular distribution is:

$$u(\alpha_\mathrm{m})=(2\cdot 10^{-6}\ °\mathrm{C}^{-1})/\sqrt{3}=1{,}15\cdot 10^{-6}\ °\mathrm{C}^{-1}$$

c) *Uncertainty of the temperature deviation $u(\theta_\mathrm{m})$*

Temperature correction is not performed . We shall consider the maximal temperature oscillation. The total uncertainty is therefore:

$$u(\theta_m)=1°\mathrm{C}/\sqrt{3}=0{,}58\ °\mathrm{C}$$

d) *Uncertainty of LI indication $u(L_\mathrm{LI})$*

Maximum permissible uncertainty for the LI is defined in accordance with the calibration history. Deviations (established by calibrations), measurement uncertainty of calibration and drift are considered. It is:

$$u(L_\mathrm{LI}) = 10\ \mathrm{nm} + 0{,}3\cdot 10^{-6}\cdot L$$

(calculation in: (calculation in: LTM_lab\4-Sistem kakovosti\Zapisi\Negotovost\ \Izračuni)

e) *Uncertainty caused by cosine error $u(e_\mathrm{cos})$*

Maximum expected value after precise positioning is 0,1 μm/m. The standard uncertainty is than

$$u(e_\mathrm{cos}) = 0{,}06\cdot 10^{-6}\cdot L$$

f) *Uncertainty caused by dead path $u(e_\mathrm{mp})$*

This component is negligible in our case.

g) *Uncertainty caused Abbe error $u(e_\mathrm{a})$*

This component is caused by different angles of *VPS* and *LI* reflector during the measurement path. The angles were measured with an electronic level and the greatest Abbe errors were evaluated to be:

- for the angle difference of 1 μm/m, Abbe error is 0,07 μm
- the greatest angle differences along the measurement path (measured with electronic level) was 342 μm/m (calculated in: *\\Fsltm5\ltm\4-Sistem kakovosti\SOP\Kalibracije\SOP 21-Črtna merila\Meritve - negotovost/Meritve - Abbe.xlsx*). Therefore, the greatest Abbe error is:

$e_a = 24\ \mu m$

Since Abbe error is corrected by angle measurements and if it is supposed that the maximum uncertainty of standard deviation in all measured points is:

$$u(e_{a1}) = 0{,}4\,\mu\text{m}$$

If it is considered that the Abbe error is corrected twice (in zero point and in measurement point), than the final result is:

$$u(e_a) = u(e_{a1}) \cdot \sqrt{2} = 0{,}6\,\mu\text{m}$$

h) Uncertainty due to the tractive force $u(e_F)$

A short test on a plastic tape has shown, that a 2 m tape segment is extended for approximately 800 µm, if the tractive force is increased for 10 N (added to the nominal force of 20 N). Additional test with 200 g and 1200 g weights has shown linear relation between the force and the extension. If relative deviation of the tractive force due to the uncertainty of the weight and friction is assumed to be within the limits of ±0,3 % (0,06 N at the nominal force of 20 N), the change in tape length would be within an interval of ±2,4 µm/m. Since the deformation interval of plastic tapes is greater than of metal tapes, it can be used for all materials. Standard uncertainty at supposed rectangular distribution is therefore:

$$u(e_F) = (2{,}4\,\mu\text{m})/\sqrt{3} = 1{,}4\,\mu\text{m/m} \text{ or } u(e_F) = 1{,}4\cdot 10^{-6}\cdot L.$$

Since exact circumstances by repositioning of the tape and the influence of the generated force on the tape extension (small wheel, ower which the tape is hanging) are not well known, the standard uncertiny is increased. The following value is put in the uncertainty budget:

$$u(e_F) = 2\cdot 10^{-6}\cdot L.$$

Table 1: Standard uncertainties of the estimations of the input values on the lower limit of measurement range (0 mm)

Value X_i	Estimated value	Standard uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
L_m	0 mm	4,71 μm	normal	1	4,71 μm
α_m	10^{-5}°C^{-1}	$1{,}15\cdot10^{-6}$ °C^{-1}	rectangular	1·°C·L	0 μm
θ_m	0 °C	0,58 °C	normal	10^{-5} °C^{-1}·L	0 μm
L_{LI}	0 mm	0,01 μm + $0{,}3\cdot10^{-6}\cdot L$	normal	1	0,01 μm
e_{cos}	0	$0{,}06\cdot10^{-6}\cdot L$	normal	1	0 μm
e_a	0	0,6 μm	rectangular	1	0,6 μm
e_F	0	$2\cdot10^{-6}\cdot L$	rectangular	1	0 μm
				Total	4,93 μm

Table 2: Standard uncertainties of the estimations of the input values on the upper limit of measurement range (200000 mm)

Value X_i	Estimated value	Standard uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
L_m	200000 mm	4,71 μm	normal	1	4,71 μm
L_{m200}	200000 mm	2,2 μm	normal	1	2,2 μm
α_m	$10\cdot10^{-6}$ °C^{-1}	$1{,}15\cdot10^{-6}$ °C^{-1}	rectangular	1·°C·L	230 μm
θ_m	0 °C	0,58 °C	normal	$10\cdot10^{-6}$ °C^{-1}·L	1160 μm
L_{LI}	200000 mm	0,01 μm + $0{,}3\cdot10^{-6}\cdot L$	normal	1	60 μm
e_{cos}	0	$0{,}06\cdot10^{-6}\cdot L$	normal	1	12 μm
e_a	0	0,6 μm	rectangular	1	0,6 μm
e_F	0	$2\cdot10^{-6}\cdot L$	rectangular	1	400 μm
				Skupno:	1249,1 μm

Combined standard uncertainty of the estimations of the input values in the best possible measurement conditions can be expressed by the equation (calculated from tables 1 and 2):

$$u = 4{,}93\ \mu m + 6{,}2\cdot10^{-6}\cdot L$$

10.3 Expanded uncertainty

Factor k=2 is used for the calculation of the expanded uncertainty. Considering the experiences and the results of international comparisons it is rounded up to:

$U = 10\ \mu\text{m} + 12{,}5 \cdot 10^{-6} \cdot L$

11 MEASUREMNET UNCERTAINTY – INCREMENTAL MS

11.1 Mathematical model of measurement

Deviation e (measurement result) is given by the expression:

$$e = L_\text{m} - L_\text{MT} + L \cdot \Delta\alpha \cdot \theta_\text{m} + L \cdot \alpha_\text{MT} \cdot \Delta T + e_\text{a} + e_F \qquad (3)$$

where:

- e - deviation (measurement result) at 20 °C
- L_m - path length between the reference position of the video probing system and the measurement position of the video probing system
- L_MT - corrected length shown by incremental measurement system
- L - nominal length
- $\Delta\alpha$ - difference between linear temperature expansion coefficient of the incremental system and measured line scale
- θ_m - temperature deviation of the measure of length from 20 °C
- α_MT - linear temperature expansion coefficient of the incremental measuring system
- ΔT - temperature difference between line scale and the incremental measuring system
- e_a - Abbe error caused by angular deviation of the video probing system
- e_F - error due to the tractive force

11.2 Standard uncertainties of the estimations of the input values and combined standard uncertainty of measurement

Equation for calculated combined standard uncertainty [3,4] has the following form:

$$u_\text{c}^2(e) = c_{L\text{m}}^2 \cdot u^2(L_\text{m}) + c_{L\text{MT}}^2 \cdot u^2(L_\text{MT}) + c_{\Delta\alpha}^2 \cdot u^2(\Delta\alpha) + c_{\theta\text{m}}^2 \cdot u^2(\theta_\text{m}) + c_{\alpha\text{MT}}^2 \cdot u^2(\alpha_\text{MT}) + \\ + c_{\Delta T}^2 \cdot u^2(\Delta T) + c_{ea}^2 \cdot u^2(e_\text{a}) + c_{eF}^2 \cdot u^2(e_F) \qquad (4)$$

Where c_i are partial derivatives of the function (3):

$c_\text{Lm} = \partial f / \partial L_\text{m} = 1$

$c_\text{LMT} = \partial f / \partial L_\text{MT} = -1$

$c_{\Delta\alpha} = \partial f / \partial \Delta\alpha = L_\text{m} \cdot \theta_\text{m} \approx L \cdot \theta_\text{m} \approx 1℃ \cdot L$ for presumed measurement conditions (CMC)

$c_{\theta\text{m}} = \partial f / \partial \theta_\text{m} = L_\text{m} \cdot \Delta\alpha \approx L \cdot \Delta\alpha \approx 12 \cdot 10^{-6} \cdot L$ for worst case

$c_{\alpha\text{MT}} = \partial f / \partial \alpha_\text{MT} = L_\text{m} \cdot \Delta T \approx L \cdot \Delta T \approx 0{,}5℃ \cdot L$ for presumed measurement conditions

$c_{\Delta\text{T}} = \partial f / \partial \Delta T = L_\text{m} \cdot \alpha_\text{MT} \approx L \cdot \alpha_\text{MT} \approx 23 \cdot 10^{-6}\,℃^{-1} \cdot L$

$c_\text{ea} = \partial f / \partial e_\text{a} = 1$

$c_{eF} = \partial f / \partial e_F = 1$

Standard uncertainties of influence values are evaluated (estimated) for the applied standard, calibration procedure, and presumed measurement conditions. In the CMC evaluation, we presume a tape measure with 4 μm repeatability, and thermal expansion coefficient of 10^{-5} °C $^{-1}$ (steel). Uncertainty of an actual calibration is evaluated by using the Excel file »Negotovost-SOP23.xlsx« (stored in the folder »SOP 23-Tračna merila«). Repeatability, and actual material of the tape measure under calibration are taken into account.

a) Uncertainty of the path length between the reference position of the video probing system and the measurement position of the video probing system u(L_m)

The uncertainty is composed of the positioning uncertainty in the reference point $(u(poz_{ref}))$ and of the positioning uncertainty in the measurement point $(u(poz_{mer}))$. It is supposed that both uncertainties are equal $(u(poz_{ref})) = (u(poz_{mer})) = (u(poz))$. Total uncertainty is:

$$u(L_m) = \sqrt{u(poz)^2 + u(poz)^2} = u(poz) \cdot \sqrt{2}$$

Standard positioning uncertainty was determined according to the resolution of the incremental system which is $1\,\mu m$. Standard deviation of these measurements that is accepted as standard uncertainty was:

$$s = (u(poz)) = 0{,}5\ \mu m;$$

Due to the quality of the lines of the tape measures 4 μm was added.

Therefore:

$$u(L_m) = 0{,}71\ \mu m + 4\ \mu m$$

Furthermore, if the measure is longer as 10 m, it should be measured in more steps. When repositioning the measure, additional positioning uncertainty appear For 200 mm, this component is:

$$u(L_{m200}) = (u(poz)) \cdot \sqrt{200/10} = 2{,}2\ \mu m$$

b) Uncertainty of MT indication u(L_{MT})

Uncertainty is composed of the uncertainty of the resolution and the uncertainty of the calibration of the measuring system.
Uncertainty of the calibration is:

$$U(cal) = 1\ \mu m + 5 \cdot 10^{-6} \cdot L\ ;\ k = 2 \quad \textit{(in-house calibration certificate)}$$

Uncertainty of the reading result in the resolution of the system $1\,\mu m$ is:

$$u(R) = 1 \mu m / \sqrt{3} = 0{,}58\ \mu m$$

The total uncertainty is:

$$u(L_{MT}) = \sqrt{u(cal)^2 + u(R)^2} = 0{,}58\ \mu m + 2{,}5 \cdot 10^{-6} \cdot L$$

c) Uncertainty of the thermal expansion coefficient difference u(Δα)

Assuming that $\alpha_{\mathrm{MT}} = 23 \cdot 10^{-6}\ °\mathrm{C}^{-1}$ and $\alpha_{\mathrm{m}} = 11 \cdot 10^{-6}\ °\mathrm{C}^{-1}$ then:

$$\Delta\alpha = \alpha_{\mathrm{m}} - \alpha_{\mathrm{MT}} = 12 \cdot 10^{-6} °\mathrm{C}^{-1}$$

Assume the maximum difference in temperature expansion $\pm 1 \cdot 10^{-6}\ °\mathrm{C}^{-1}$. The standard uncertainty is than:

$$u(\Delta\alpha\ ) = (2 \cdot 10^{-6}\ °\mathrm{C}^{-1}) / \sqrt{3} = 1{,}15 \cdot 10^{-6}\ °\mathrm{C}^{-1}$$

d) Uncertainty of the temperature deviation u(θ_m)

Temperature correction is not performed. We shall consider the maximal temperature oscillation $\pm 1 °\mathrm{C}$. The total uncertainty is therefore:

$$u(\theta_{\mathrm{m}}) = 1 °\mathrm{C} / \sqrt{3} = 0{,}58\ °\mathrm{C}$$

e) Uncertainty of linear temperature expansion coefficient of the incremental system u(α_{MT})

Assume that the error limits of the possible thermal expansion is $\pm 0{,}5 \cdot 10^{-6}\ °\mathrm{C}^{-1}$.

Standard uncertainty at supposed rectangular distribution is:

$$u(\alpha_{\mathrm{MT}}) = \left(0{,}5 \cdot 10^{-6}\ °\mathrm{C}^{-1}\right) / \sqrt{3} = 0{,}29 \cdot 10^{-6} °\mathrm{C}^{-1}$$

f) Uncertainty of the temperature difference u(ΔT)

Based on temperature measurements, it can be assumed that the temperature difference is in the interval $\pm 0{,}5\ °\mathrm{C}$.

Standard uncertainty at supposed rectangular distribution is:

$$u(\Delta T) = (0{,}5\ °\mathrm{C}) / \sqrt{3} = 0{,}3\ °\mathrm{C}$$

g) Uncertainty caused Abbe error u(e_a)

This component is caused by different angles of *VPS* and *LI* reflector during the measurement path. The angles were measured with an electronic level and the greatest Abbe errors were evaluated to be:

- for the angle difference of 1 μm/m, Abbe error is 0,07 μm
- the greatest angle differences along the measurement path (measured with electronic level) was 342 μm/m (calculated in: *\\Fsltm5\ltm\4-Sistem kakovosti\SOP\Kalibracije\SOP 21-Črtna merila\Meritve - negotovost/Meritve - Abbe.xlsx*). Therefore, the greatest Abbe error is:

$$e_{\mathrm{a}} = 24\ \mu\mathrm{m}$$

Since Abbe error is corrected by angle measurements and if it is supposed that the maximal uncertainty of standard deviation in all measured points is (*calculated in: \\Fsltm5\ltm\4-Sistem kakovosti\SOP\Kalibracije\SOP 23-Tračna merila\Meritve - negotovost/Meritve - Abbe.xlsx)*::

$$u(e_{a1}) = 0{,}4\,\mu\text{m}$$

If it is considered that the Abbe error is corrected twice (in zero point and in measurement point), than the final result is:

$$u(e_a) = \sqrt{u(e_{a1})^2 + u(e_{a1})^2} = u(e_{a1})\cdot\sqrt{2} = 0{,}6\,\mu\text{m}$$

h) ***Uncertainty due to the tractive force u(e_F)***

A short test on a plastic tape has shown, that a 2 m tape segment is extended for approximately 800 μm, if the tractive force is increased for 10 N (added to the nominal force of 20 N). Additional test with 200 g and 1200 g weights has shown linear relation between the force and the extension. If relative deviation of the tractive force due to the uncertainty of the weight and friction is assumed to be within the limits of ±0,3 % (0,06 N at the nominal force of 20 N), the change in tape length would be within an interval of ±2,4 μm/m. Since the deformation interval of plastic tapes is greater than of metal tapes, it can be used for all materials. Standard uncertainty at supposed rectangular distribution is therefore:

$u(e_F) = (2{,}4\,\mu\text{m})/\sqrt{3} = 1{,}4\,\mu\text{m/m}$ or $u(e_F) = 1{,}4\cdot10^{-6}\cdot L$

Since exact circumstances by repositioning of the tape and the influence of the generated force on the tape extension (small wheel, ower which the tape is hanging) are not well known, the standard uncertainy is increased. The following value is put in the uncertainty budget:

$u(e_F) = 2\cdot10^{-6}\cdot L$

Table 3: Standard uncertainties of the estimations of the input values on the lower limit of measurement range (0 mm)

Value X_i	Estimated value	Standard uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
L_m	0 mm	4,71 μm	normal	1	4,71 μm
L_{MT}	0 mm	$0{,}58\ \mu m + 2{,}5\cdot10^{-6}\cdot L$	normalna	1	0,58 μm
$\Delta\alpha$	$12\cdot10^{-6}\ °C^{-1}$	$1{,}15\cdot10^{-6}\ °C^{-1}$	rectangular	$1\cdot °C\cdot L$	0 μm
θ_m	0 °C	0,58 °C	rectangular	$12\cdot10^{-6}\ °C^{-1}\cdot L$	0 μm
α_{MT}	$23\cdot10^{-6}\ °C^{-1}$	$0{,}29\cdot10^{-6}$	rectangular	$0{,}5\ °C\cdot L$	0 μm
ΔT	0,5 °C	0,3 °C	rectangular	$23\cdot10^{-6}\ °C^{-1}\cdot L$	0 μm
e_a	0	0,6 μm	normal	1	0,6 μm
e_F	0	$2\cdot10^{-6}\cdot L$	rectangular	1	0 μm
				Total:	4,78 μm

Table 4: Standard uncertainties of the estimations of the input values on the upper limit of measurement range (200000 mm)

Value X_i	Estimated value	Standard uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
L_m	200000 mm	4,71 μm	normal	1	4,71 μm
L_{m200}	200000 mm	2,2 μm	normal	1	2,2 μm
L_{MT}	200000 mm	$0{,}58\ \mu m + 2{,}5\cdot10^{-6}\cdot L$	normalna	1	500,6 μm
$\Delta\alpha$	$12\cdot10^{-6}\ °C^{-1}$	$1{,}15\cdot10^{-6}\ °C^{-1}$	rectangular	$1\cdot °C\cdot L$	230 μm
θ_m	0 °C	0,58 °C	rectangular	$12\cdot10^{-6}\ °C^{-1}\cdot L$	1392 μm
α_{MT}	$23\cdot10^{-6}\ °C^{-1}$	$0{,}29\cdot10^{-6}$	rectangular	$0{,}5\ °C\cdot L$	29 μm
ΔT	0,5 °C	0,3 °C	rectangular	$23\cdot10^{-6}\ °C^{-1}\cdot L$	1380 μm
e_a	0	0,6 μm	normal	1	0,6 μm
e_F	0	$2\cdot10^{-6}\cdot L$	rectangular	1	400 μm
				Total:	2075 μm

Combined standard uncertainty of the estimations of the input values in the best possible measurement conditions can be expressed by the equation (calculated from tables 3 and 4):

$$u = 4{,}8\ \mu m + 10{,}4\cdot10^{-6}\cdot L$$

11.3 Expanded uncertainty

Factor k=2 is used for the calculation of the expanded uncertainty. Considering the experiences and the results of international comparisons it is rounded up to:

$U = 10\ \mu m + 22{,}5 \cdot 10^{-6} \cdot L$

12 TRACEABILITY

Measurement equipment used for calibration:

- Incremental measurement system - calibrated in LTM.
- Video probing system - no calibration necessary (only positioning)
- Laser interferometer (*LI*) - calibrated in LTM
- weights 0,5 kg, 1 kg, 2 kg in 5 kg - checked by weighing at FS

13 LITERATURE

[1] Laser Measurement System 5528A. User's Guide Hewlett Packard

[2] JCGM 100:2008 - Evaluation of measurement data — Guide to the expression of uncertainty in measurement

[3] EA 4/02 M: 2013: Evaluation of the Uncertainty of Measurement in Calibration